# 등가 자기회로 및 자기에너지

1. 등가자기회로법 (equivalent magnetic circuit method)

2. 등가자기회로법과 유한요소법
   (equivalent magnetic circuit method and finite element method)

3. 쇄교자속과 인덕턴스 (flux linkage and inductance)

4. 자기에너지 및 자기수반에너지 (magnetic energy and magnetic coenergy)

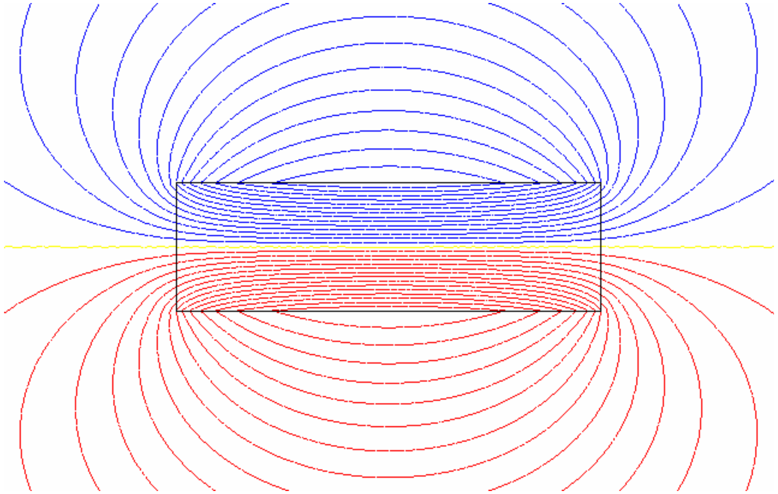
5. 힘, 토크, 파워 (force, torque, power)

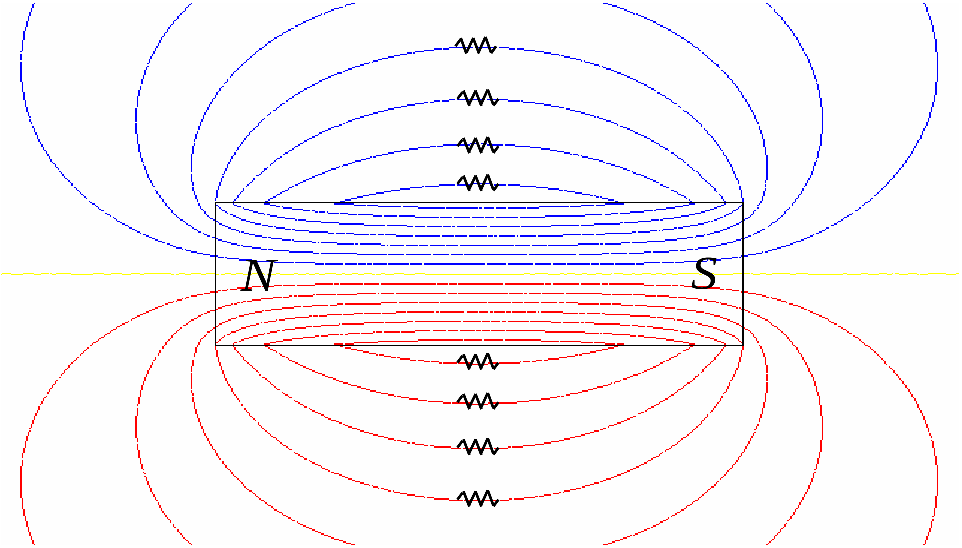
## 1. 등가자기회로법

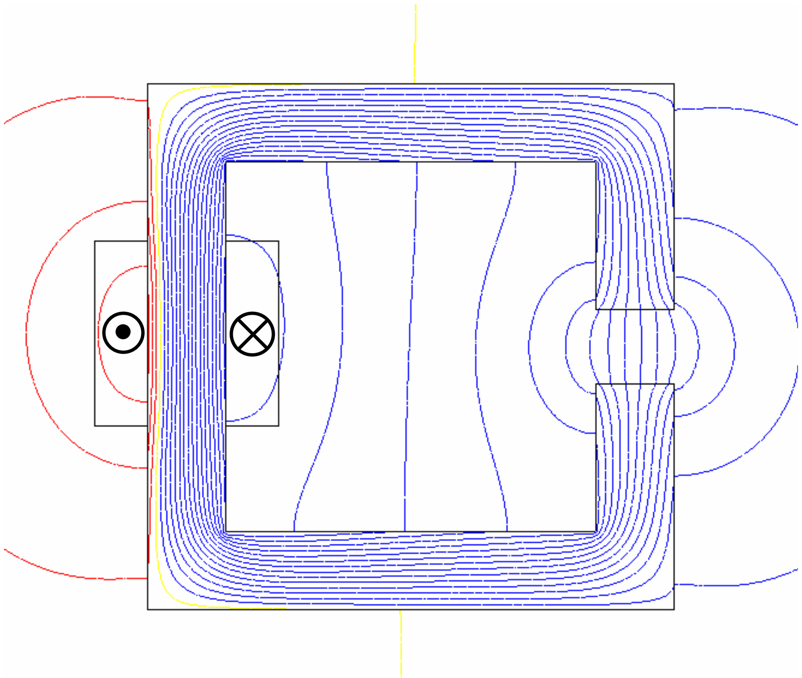
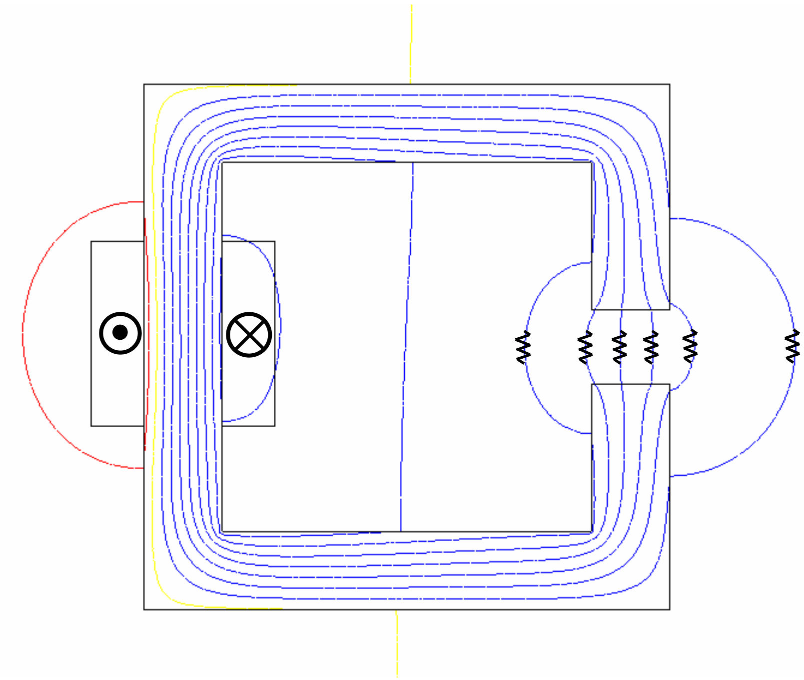
■ 전동기 특성해석 방법

1. 해석적인 방법 (analytical method)

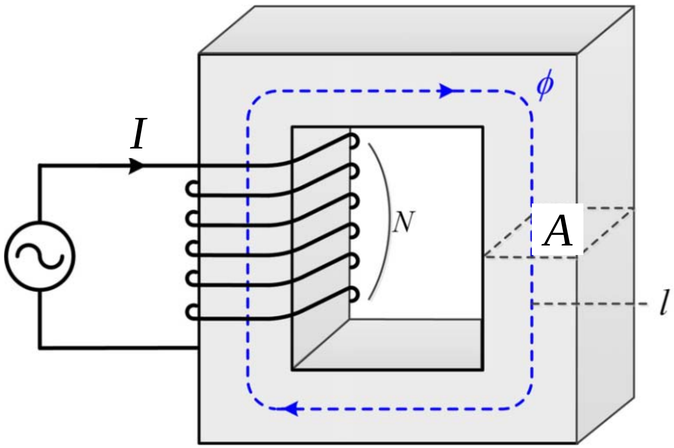
2. 수치해석적인 방법 (numerical method)

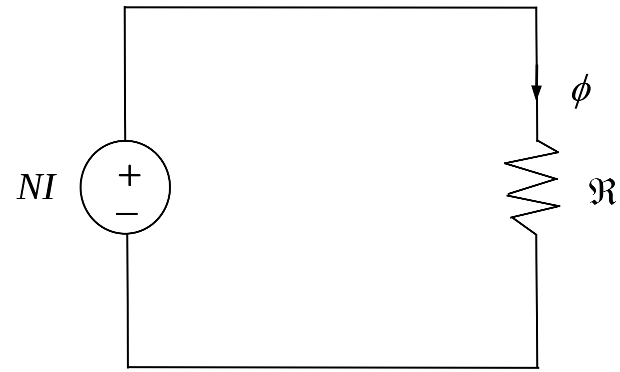



N
S

## (1) 1권선 자기회로 (singly excited magnetic structure)



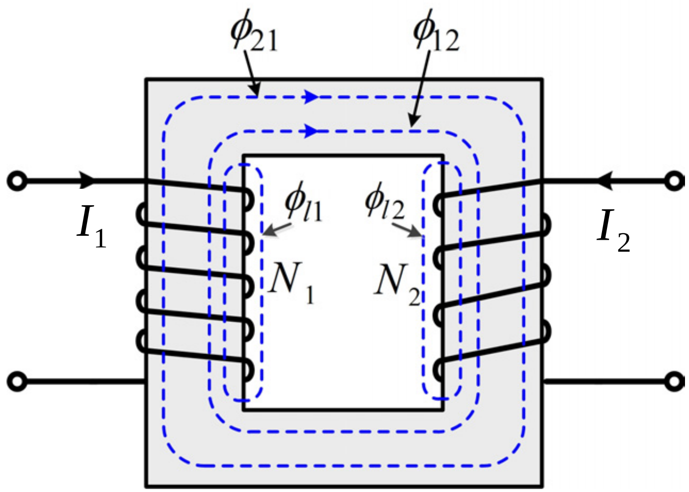



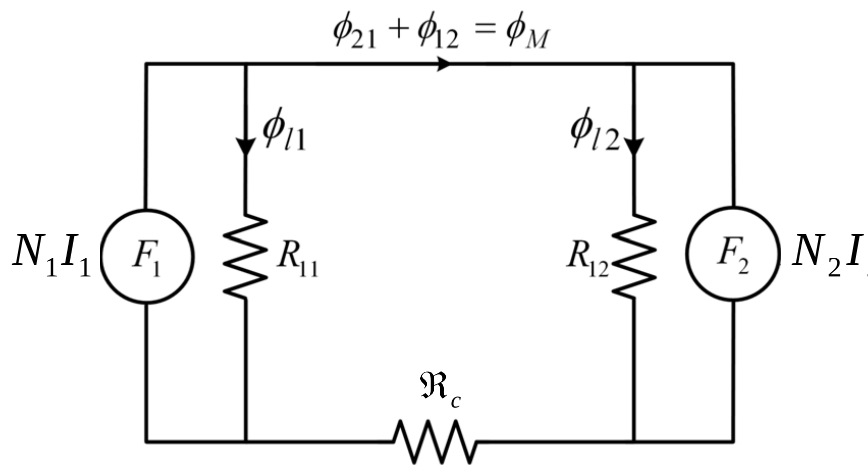

$$F = \oint \vec{H} \cdot d\vec{l} = NI \qquad \phi = \int_s \vec{B} \cdot d\vec{S} = BA$$

$$F = Hl = \frac{B}{\mu} l = \frac{\phi}{\mu A} l$$

$$\phi = \frac{\mu A}{l} F$$

## (2) 2권선 자기회로 (doubly excited magnetic structure)




전류$I_1$ 에 의해 권선 1을 통과하는 자속량 : $\phi_{11}$

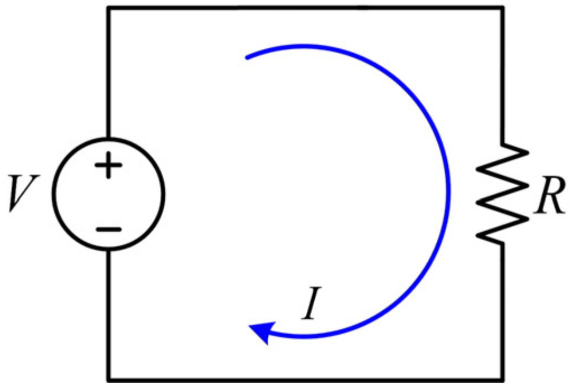
$$\phi_{11} = \phi_{l1} + \phi_{21}$$

전류$I_2$ 에 의해 권선 1을 통과하는 자속량 : $\phi_{22}$

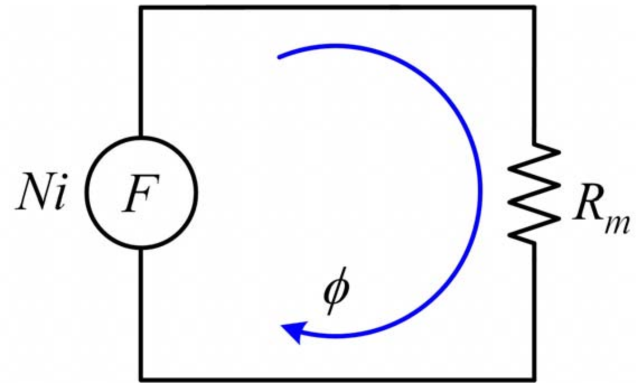
$$\phi_{22} = \phi_{12} + \phi_{l2}$$

※ $\phi_{ab}$ : b권선에 의해 발생한 자속이 a권선을 통과하는 자속

(3) 전기회로와의 상사성 (duality with electric circuit)



(a) 전기회로



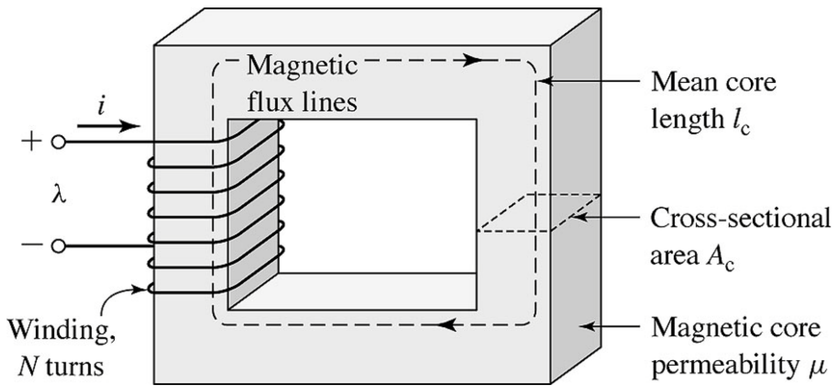
(b) 자기회로

| 전기회로 | 자기회로 |
|---|---|
| $V = Ri$ | |
| $V$ : emf [V] (electromotive force; 기전력) | |
| $i$ : 전류 [A] | |
| $R$ : 저항 [Ω] | |
| 키르히호프의 전류법칙(KCL) | |
| 키르히호프의 전압법칙(KVL) | |

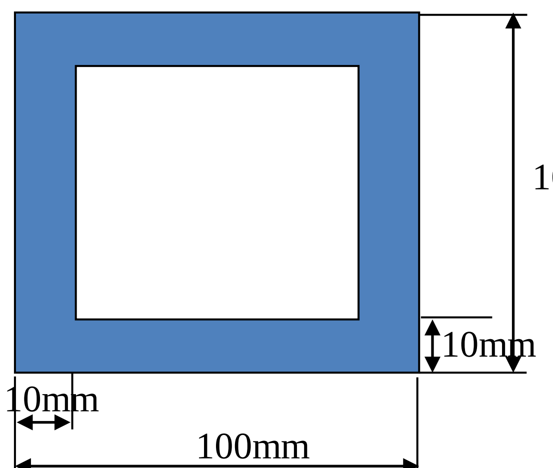
## 2. 등가자기회로법과 유한요소법

### (1) 공극이 없는 경우

#### 1) 유한요소법

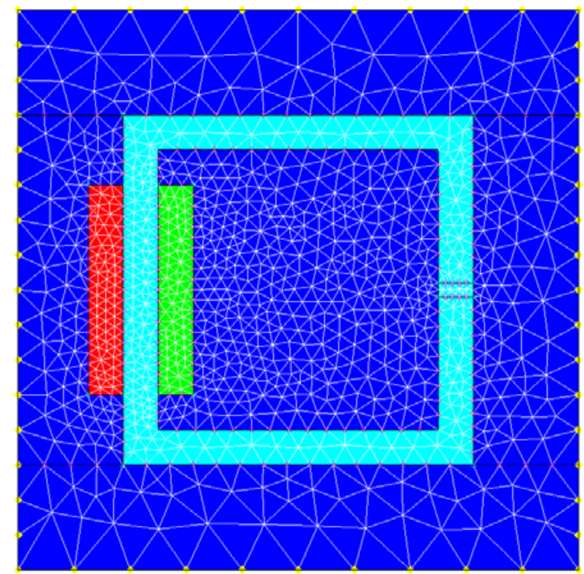



턴수 : 10턴,  인가전류 : 100A




Depth : 10mm

해석모델



유한요소모델

1. 선형해석

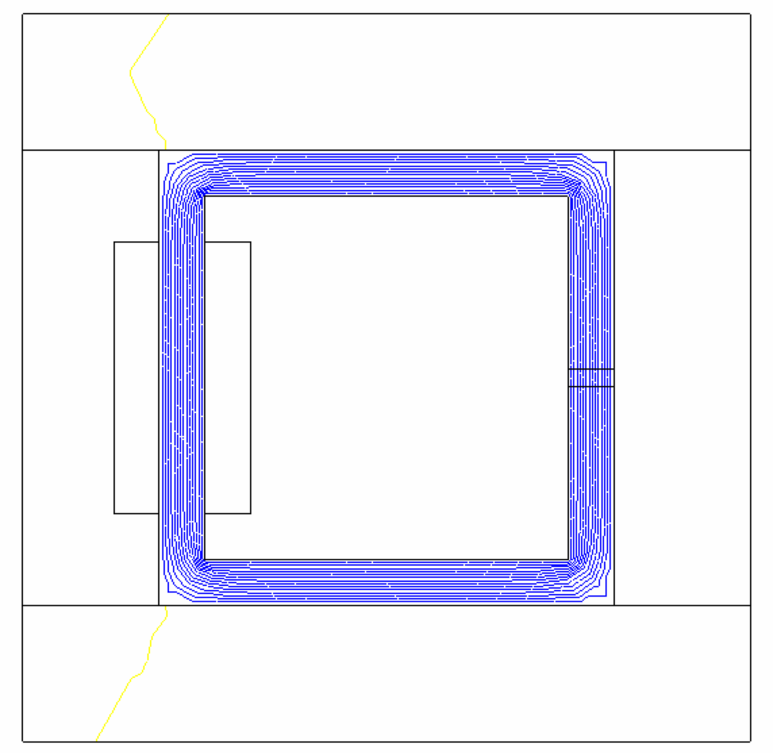
비투자율 : 3000

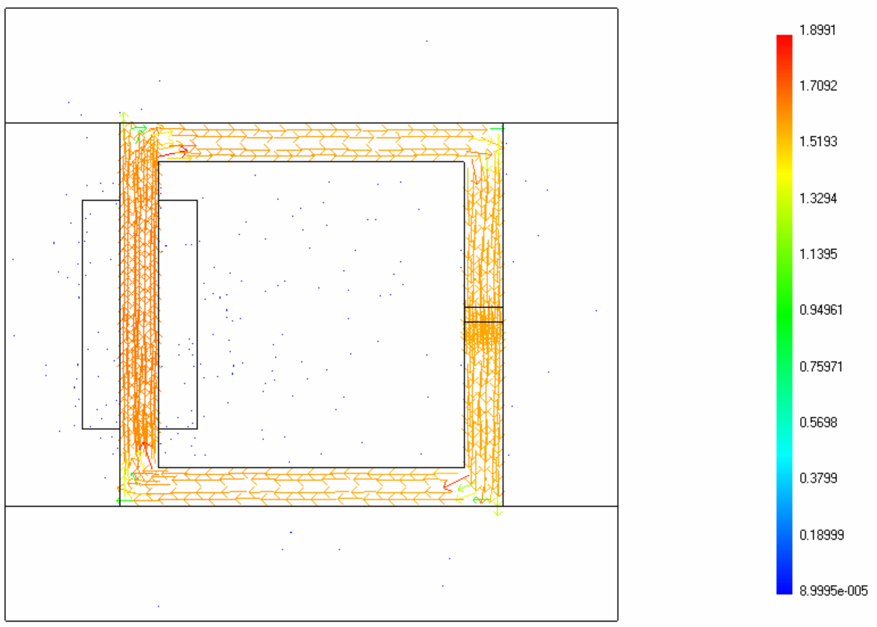
- Magnetic energy : 54.82 (J)
- Magnetic co-energy : 54.82 (J)
- Flux : $1.09 \times 10^{-3}$ (wb)

2. 비선형해석

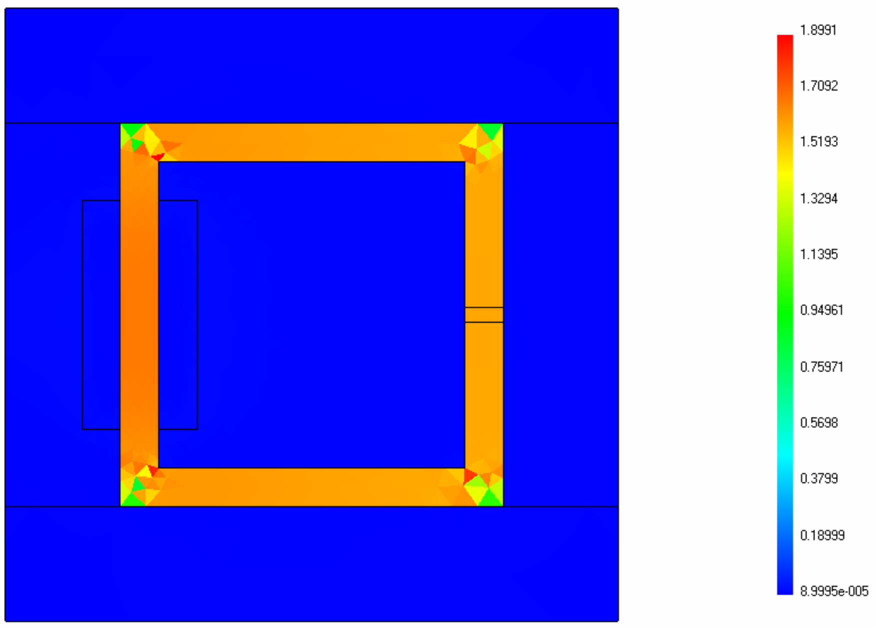
재질 : S18

- Magnetic energy : 1.88 (J)
- Magnetic co-energy : 14.81 (J)
- Flux linkage : $1.66 \times 10^{-4}$ (wb)



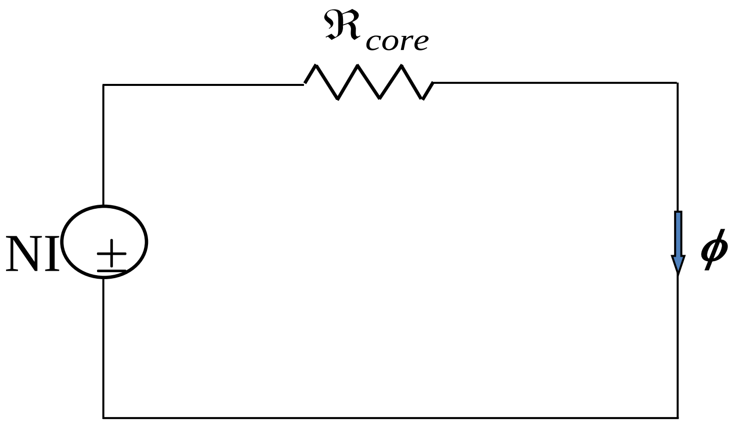

1.8991
1.7092
1.5193
1.3294
1.1395
0.94961
0.75971
0.5698
0.3799
0.18999
8.9995e-005

벡터선도


1.8991
1.7092
1.5193
1.3294
1.1395
0.94961
0.75971
0.5698
0.3799
0.18999
8.9995e-005

자속밀도분포

2) 등가자기회로법

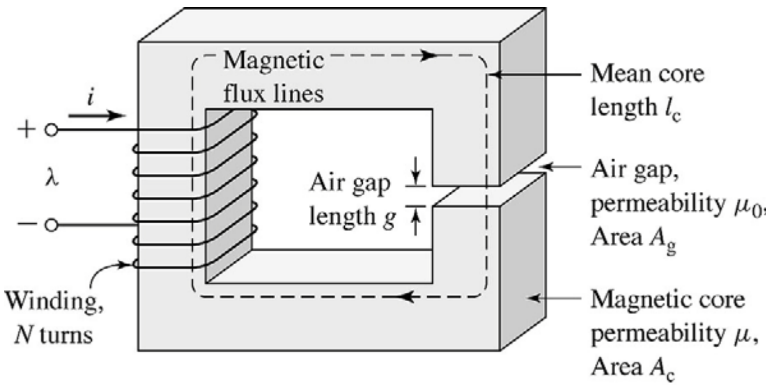


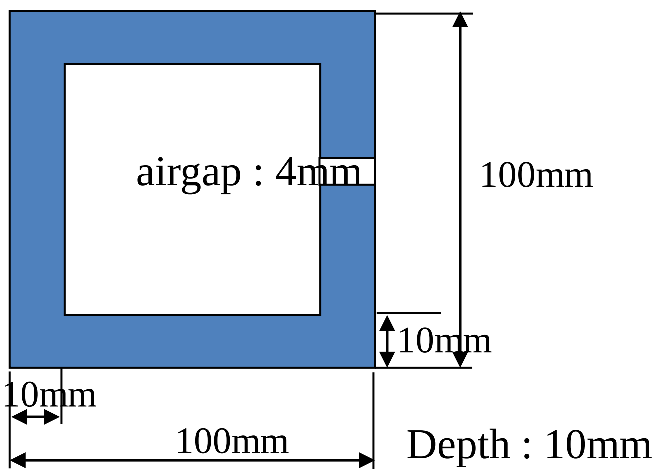

$$\therefore \phi = \frac{F}{\Re_{core}}$$

$$= \frac{1000}{954929} = 1.05 \times 10^{-3}\,(wb)$$

# (2) 공극이 있는 경우

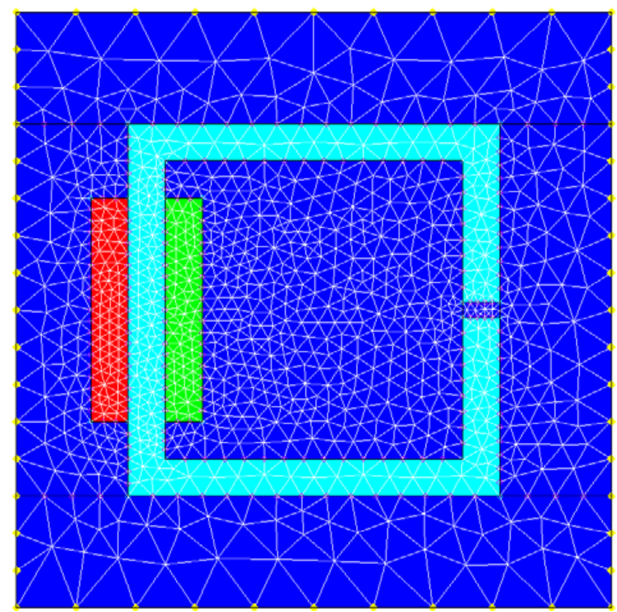
## 1) 유한요소법




해석모델

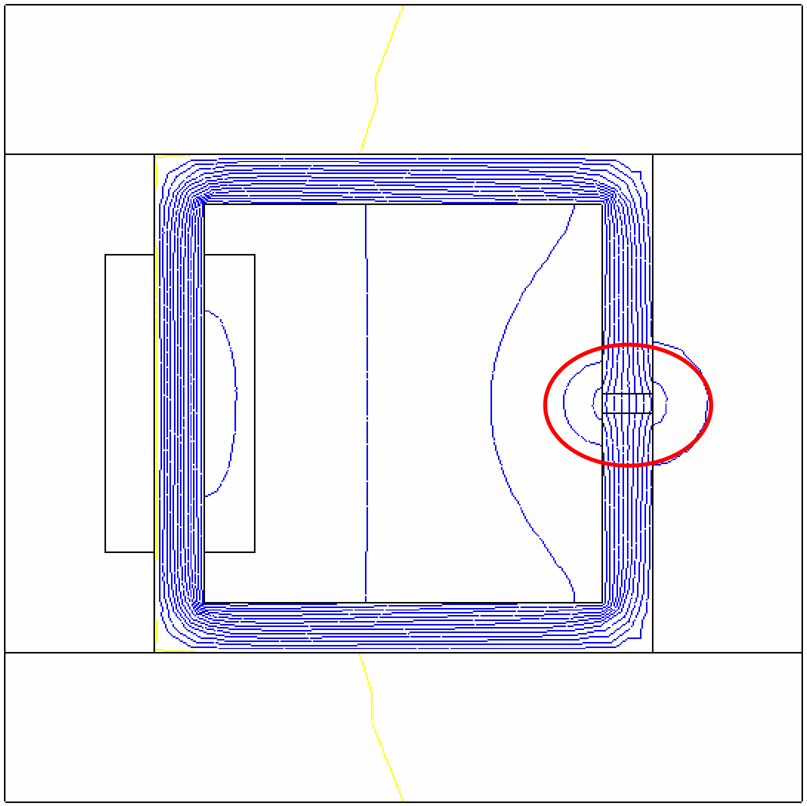
턴수 : 10턴,   인가전류 : 100A



1. 선형해석

비투자율 : 3000

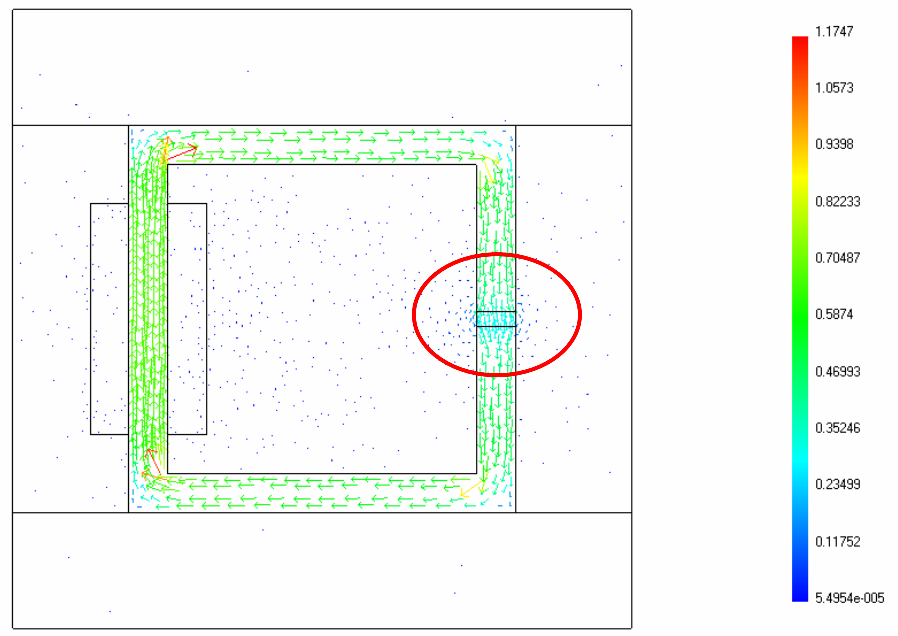
- Magnetic energy : 3.27 (J)
- Magnetic co-energy : 3.27 (J)
- Flux : $6.54 \times 10^{-5}$ (wb)



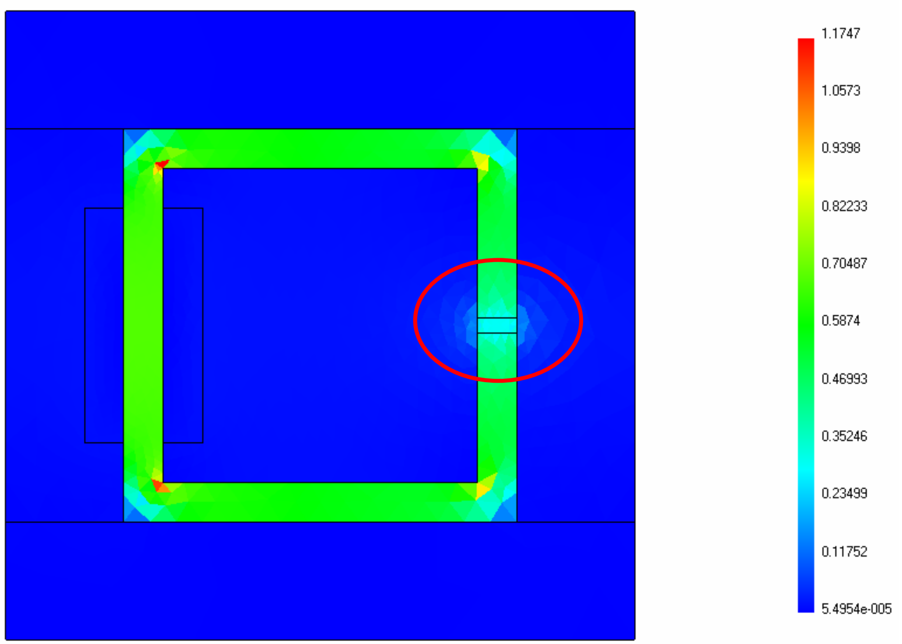
2. 비선형해석

재질 : S18

- Magnetic energy : 3.34 (J)
- Magnetic co-energy : 3.32 (J)
- Flux linkage : 6.66×10-5 (wb)

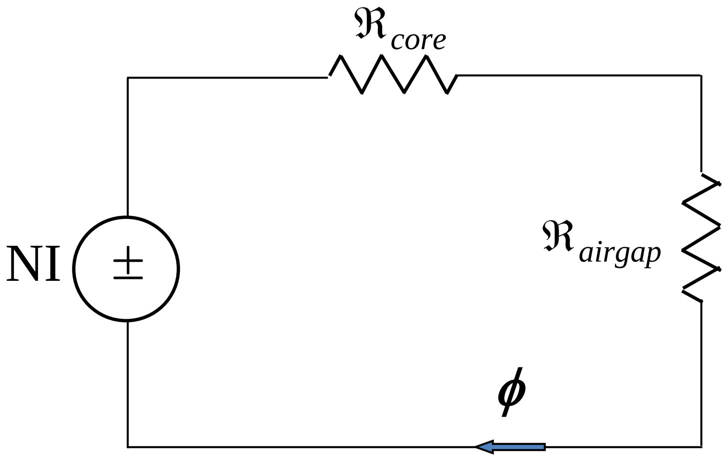

1.1747
1.0573
0.9398
0.82233
0.70487
0.5874
0.46993
0.35246
0.23499
0.11752
5.4954e-005

벡터선도


1.1747
1.0573
0.9398
0.82233
0.70487
0.5874
0.46993
0.35246
0.23499
0.11752
5.4954e-005

자속밀도분포

2) 등가자기회로법




$$\therefore \phi = \frac{F}{\Re_{core} + \Re_{airgap}}$$

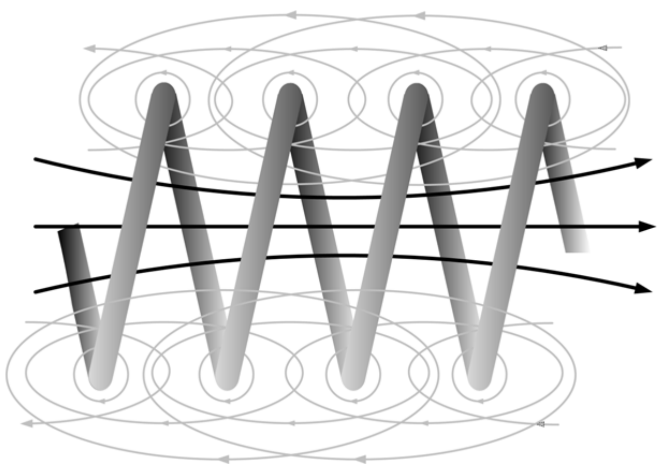
$$= \frac{1000}{9.44 \times 10^{5} + 3.18 \times 10^{7}} = 3.05 \times 10^{-5}\ (wb)$$

# 3. 쇄교자속과 인덕턴스

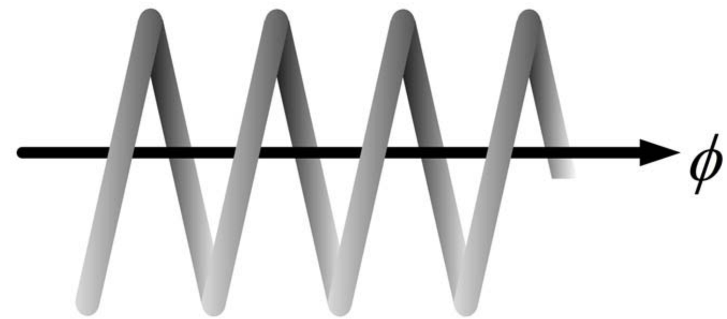
## (1) 쇄교자속

$$\left(N\phi\right)_{total} = \lambda_1 + \lambda_2 + \cdots + \lambda_i + \cdots + \lambda_N \ [\mathrm{Wb}]$$

$$\lambda = N\phi \ [\mathrm{Wb}]$$



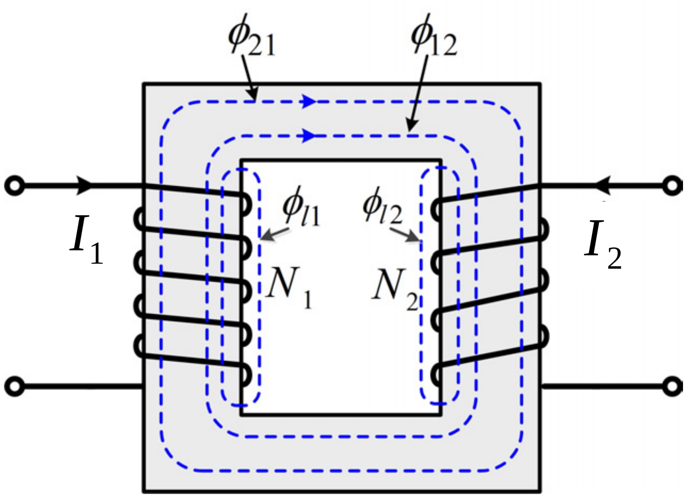
(a) 부분적으로 쇄교하는 자속을 고려한 모델



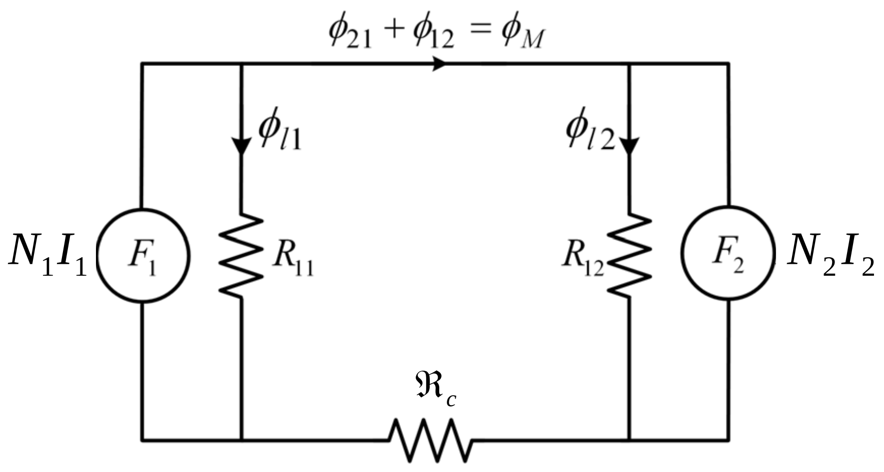

(b) 일반적인 쇄교자속모델

• **상호자속**

• **누설자속**




(a) 자성체 코어에서의 자속흐름




(b) 누설을 고려한 자기등가회로

- 전류 1에 의해 권선 1을 쇄교하는 자속 : 권선1의 자기쇄교자속

$$\lambda_{11} = N_1\phi_{11} = N_1(\phi_{l1} + \phi_{21}) = N_1\left(\frac{N_1 I_1}{\Re_{l1}} + \frac{N_1 I_1}{\Re_c}\right) [wb]$$

- 전류 2에 의해 권선 2를 쇄교하는 자속 : 권선2의 자기쇄교자속

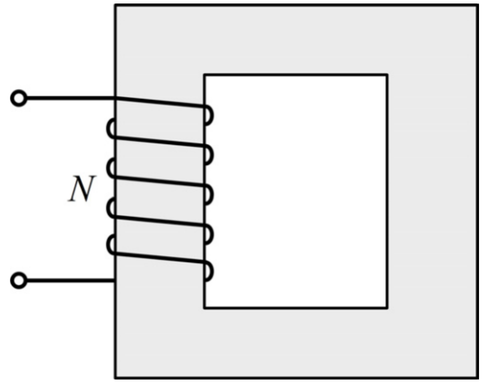
$$\lambda_{22} = N_2\phi_{22} = N_2(\phi_{12} + \phi_{l2}) = N_2\left(\frac{N_2 I_2}{\Re_c} + \frac{N_2 I_2}{\Re_{l2}}\right) [wb]$$

- 상호쇄교자속

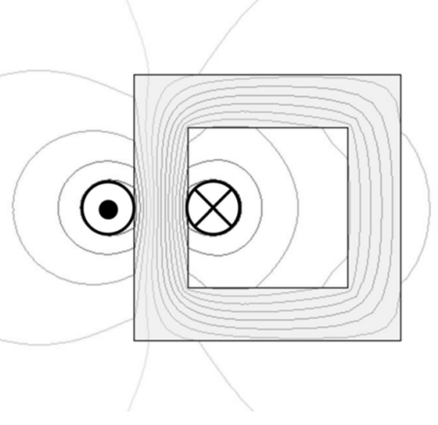
$$\lambda_{12} = N_1\phi_{12} = N_1 \frac{N_2 I_2}{\Re_c} [wb]$$

$$\lambda_{21} = N_2\phi_{21} = N_2 \frac{N_1 I_1}{\Re_c} [wb]$$

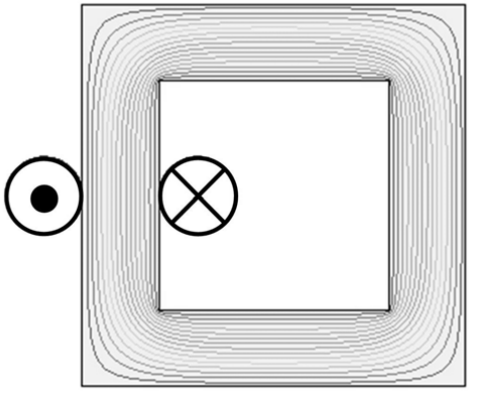
## ❖ 누설자속의 성질




투자율과 누설자속의 연관성을 알아보기 위한 모델



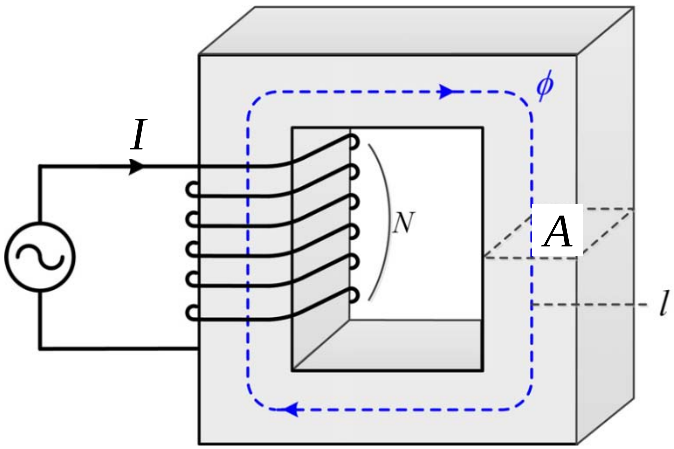
$\mu_r = 10$



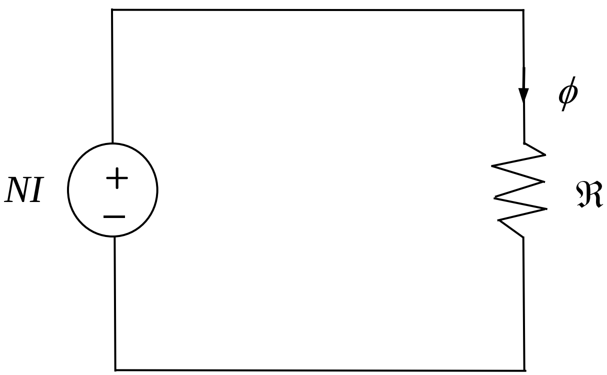
$\mu_r = 1{,}000$

## (2) 인덕턴스

### 1) 1권선 자기회로 (singly excited magnetic structure)




<single excited magnetic structure>



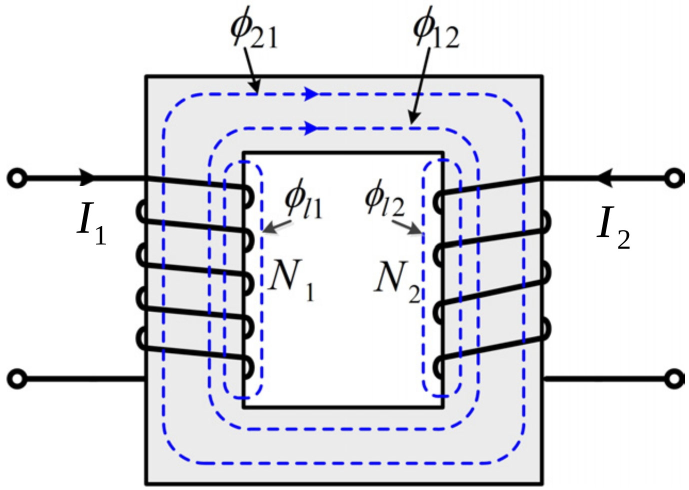

<equivalent magnetic circuit>

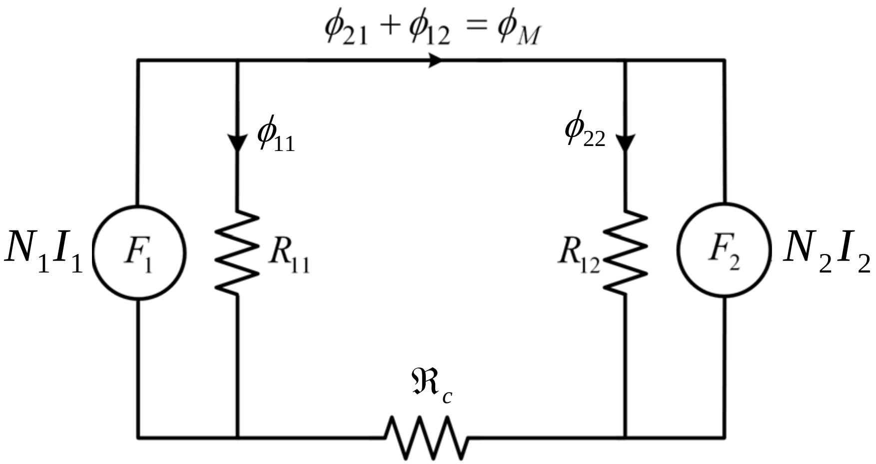
$$\phi = \frac{NI}{\Re}$$

$$\lambda = N\phi$$

$$\lambda = \frac{N^2}{\Re}\ \mathrm{I}$$

## 2) 2권선 자기회로 (doubly excited magnetic structure)




- **쇄교자속량**

$$\lambda_1 = N_1\phi_1 = N_1(\phi_{11}+\phi_{12}) = N_1\left(\frac{N_1I_1}{\Re_{l1}}+\frac{N_1I_1}{\Re_c}\right)+N_1\frac{N_2I_2}{\Re_c}\ [wb]$$

$$= L_{11}I_1 + L_{12}I_2$$

$$\lambda_2 = N_2\phi_2 = N_2(\phi_{21}+\phi_{22}) = N_2\frac{N_1I_1}{\Re_c}+N_2\left(\frac{N_2I_2}{\Re_{l2}}+\frac{N_2I_2}{\Re_c}\right)\ [wb]$$

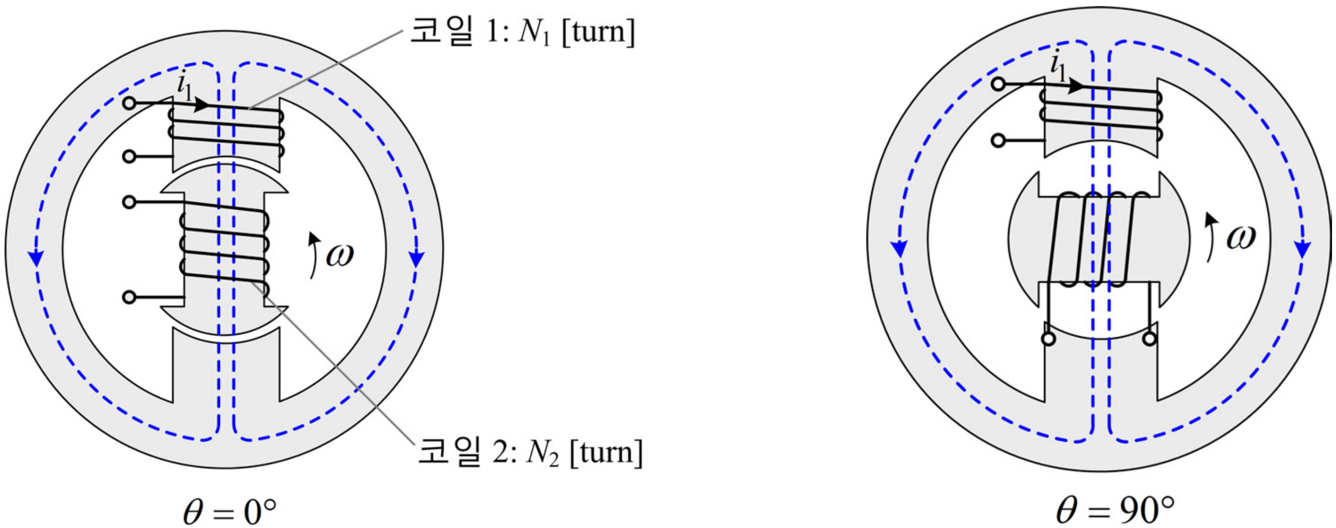
$$= L_{21}I_1 + L_{22}I_2$$

- **자기인덕턴스 (self inductance)**

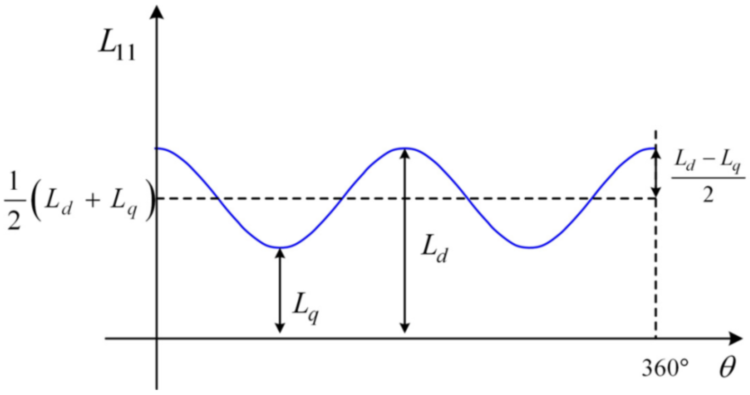
- **상호인덕턴스 (mutual inductance)**

$$\begin{cases} \Re_c = 0 \\ \\ \Re_c = \infty \end{cases}$$

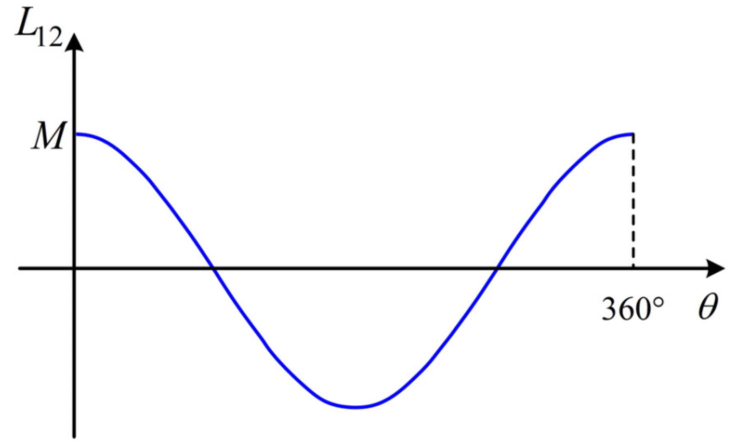
## (3) 회전기에서의 자속과 인덕턴스




**상대운동에 따른 상호 인덕턴스의 변화**




위치에 따른 자기인덕턴스 변화




위치에 따른 상호인덕턴스 변화

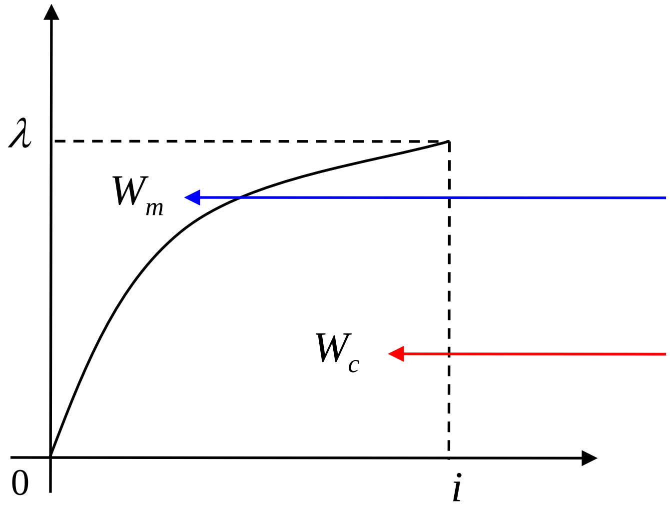
## 4. 자기에너지, 자기수반에너지 (magnetic energy, magnetic coenergy)

### (1) 1권선 회로에서의 자기에너지 및 자기수반에너지

$$p = e\,i = i\,\frac{d\lambda}{dt}$$

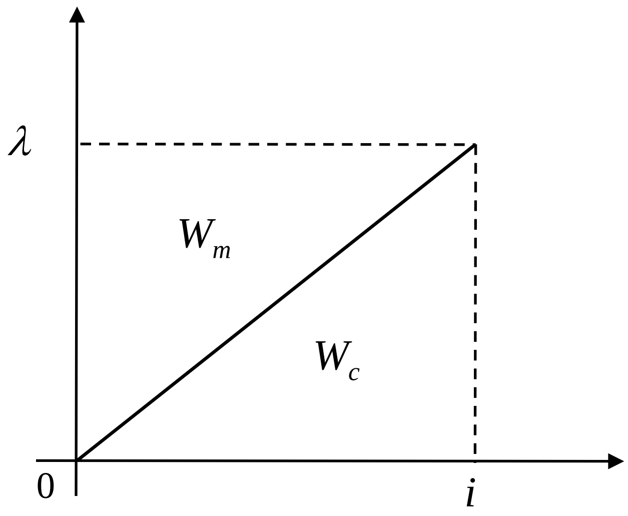
$$W_m = \int_0^t i\,\frac{d\lambda}{dt}\,dt = \int_{\lambda(0)}^{\lambda(t)} i\,d\lambda$$

$$\begin{pmatrix} \lambda(0) : \text{initial flux linkage} \\ \lambda(t) : \text{flux linkage at time } t \end{pmatrix}$$




$$W_m = \int i(\lambda, x)\,d\lambda$$

$$W_c = \int \lambda(i, x)\,di$$




If $\lambda = L(x) \cdot i$

- Linear system
- $W_m = W_c$

$$W_m = \int_{\lambda(0)}^{\lambda(t)} i\, d\lambda = \int_{\lambda(0)}^{\lambda(t)} \frac{\lambda}{L} d\lambda = \frac{1}{2L}\left[\lambda(t)^2 - \lambda(0)^2\right][J]$$

$$\lambda(0) = 0 \quad , \quad \lambda(t) = \lambda \quad \Rightarrow$$

자속량보다는 전류량으로 에너지를 표현하는 것이 편리

=> 자기수반에너지 (magnetic coenergy)

$$W_c = \int_{i(0)}^{i(t)} \lambda\, di = \int_{i(0)}^{i(t)} Li\, di = \frac{L}{2}\left[i(t)^2 - i(0)^2\right][J]$$

$$i(0) = 0 \quad , \quad i(t) = i \quad \Rightarrow$$

- $P = \dfrac{\mu A}{l}$ , $\lambda = N\phi$ , $L = N^2 P$ and $F = Ni$

$$W_m = \frac{\lambda^2}{2L} = \frac{(N\phi)^2}{2(N^2 P)} = \frac{\phi^2}{2P} = \frac{F^2}{2\Re}(J)$$

$$W_c = \frac{1}{2}Li^2 = \frac{1}{2}(N^2 P)i^2 = \frac{1}{2}PF^2 = \frac{1}{2}\Re\phi^2(J)$$

- $\phi = BA$ , $F = Hl$ and volume is $Al$

$$w_m = \frac{W}{Al} = \frac{\phi^2}{2PAl} = \frac{(BA)^2}{2(\mu A/l)Al} = \frac{B^2}{2\mu}\left(J/m^3\right)$$

$$w_c = \frac{W_c}{Al} = \frac{1}{2Al}PF^2 = \frac{1}{2Al}\frac{\mu A}{l}(Hl)^2 = \frac{\mu H^2}{2}\left(J/m^3\right)$$

## (2) 2권선 회로에서의 자기에너지 및 자기수반에너지

$$p = i_1 \frac{d\lambda_1}{dt} + i_2 \frac{d\lambda_2}{dt}$$

$$W_m = \frac{\lambda_{11}{}^2}{2L_1} + \frac{\lambda_{22}{}^2}{2L_2} + \frac{\lambda_{12}\lambda_{21}}{L_{12}}$$

$$\left( \begin{array}{l} \lambda_{11} = N_1\phi_{11} \ , \ \lambda_{22} = N_2\phi_{22} \\ \lambda_{12} = N_1\phi_{12} \ , \ \lambda_{21} = N_2\phi_{21} \end{array} \right)$$

$$W_c = \frac{1}{2}L_1 i_1{}^2 + \frac{1}{2}L_2 i_2{}^2 + i_1 i_2 L_{12}$$

# 5. 힘, 토크, 파워

## (1) 기본관계 (Basic Relationships)

- **선형 시스템 (translational system)**

$$dW_{me} = F\,dx\ [J]$$

$$P_{me} = \frac{dW_{me}}{dt} = F\frac{dx}{dt} = F\,v\ [W]$$

- **회전형 시스템 (rotational system)**

$$dW_{me} = T\,d\theta\ [\mathrm{J}]$$

$$P_{me} = \frac{dW_{me}}{dt} = T\frac{d\theta}{dt} = T\,\omega[W]$$

## (2) Fundamental Implications

$$T = kD^2 L$$

$$P_m = T\,\boldsymbol{\omega}$$

$$T \propto D^2 \quad , \quad P_m \propto T \quad \Rightarrow \quad P_m \propto D^2$$

=> 지름 증가하면 토크, 및 기계적 출력이 증가

① volume ∝ $D^2$ →

② mass ∝ volume ∝ $D^2$

③ inertia of rotor ∝ $D^4$

④ power rate ∝ $\dfrac{T^2}{J}$ →